

Relativistic anisotropic charged fluid spheres with varying cosmological constant

Saibal Ray^{1*} †, Sumana Bhadra² and Anand S. Sengupta^{1‡}

¹Inter-University Centre for Astronomy and Astrophysics, Post Box 4, Pune 411 007, India

² Balichak Girls' High School, Balichak, West Midnapur 721 124, West Bengal, India

Abstract. Static spherically symmetric anisotropic source has been studied for the Einstein-Maxwell field equations assuming the erstwhile cosmological constant Λ to be a space-variable scalar, viz., $\Lambda = \Lambda(r)$. Two cases are examined out of which one reduces to isotropic sphere. The solutions thus obtained are shown to be electromagnetic in origin as a particular case. It is also shown that the generally used pure charge condition, viz., $\rho + p_r = 0$ is not always required for constructing electromagnetic mass models.

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1. Introduction

The cosmological constant Λ , related to the energy of space, introduced by Einstein in general relativity has become of late very significant from the view point of cosmology. Though Einstein ultimately abandoned it stating that it was a “blunder” in his life but Tolman keeps it as a constant quantity in his field equations even in 1939’s famous work related to astrophysical system (Tolman, 1939). It is also, in favor of keeping Λ , argued by Peebles and Ratra (2002) that like all energy, the zero-point energy related to space has to contribute to the source term in Einstein’s gravitational field equations. However, it is being gradually felt that the erstwhile cosmological constant Λ is indeed a scalar variable dependent on time rather than a constant as was being believed earlier. Recently, this variation in cosmological constant is also observationally confirmed due to the evidence of high redshift Type Ia supernova (Perlmutter *et al.*, 1998; Riess *et*

*Permanent address: Department of Physics, Barasat Government College, Barasat 743 201, West Bengal, India

†saibal@iucaa.ernet.in

‡anandss@iucaa.ernet.in

al., 1998) for a small decreasing value which is $\leq 10^{-56} cm^{-2}$ at the present epoch. Obviously, once the Λ becomes a scalar its dependence need not be limited only to time coordinate (as in cosmology). Since it enters in the field equations as a variable, it must be dependent on space coordinates as well. Therefore, in general, the Λ is a scalar variable dependent on, either or both, space and time coordinates. We believe that just as in cosmology the dependence of Λ on time has been found to be of vital importance playing a significant role now, its dependence on space coordinates is equally important for astrophysical problems, in particular, the problems of small dimensions like that of an extended electron. With this view point, we consider here a charged anisotropic static spherically symmetric fluid source of physical radius, a , by introducing a scalar variable Λ dependent on the radial coordinate. The field equations thus obtained, under certain mathematical assumptions, yield a set of solutions which has a kind of another historical importance, known in the name of Electromagnetic Mass Models (EMMM) in the literature (Feynman *et al.*, 1964). The effective gravitational mass of these models depends on the electromagnetic field alone (the effective gravitational mass vanishes when the charge density vanishes). Such models have been studied by several authors (Tiwari *et al.* 1984, 1986, 1991; Gautreau, 1985; Grøn, 1985, 1986a, 1986b; Ponce de Leon, 1987a, 1987b, 1988; Tiwari and Ray, 1991a, 1991b, 1997; Ray *et al.*, 1993; Ray and Ray, 1993). All these EMMM, however, have been obtained under a special assumption $\rho + p = 0$, where ρ is the matter-energy density and p is the fluid pressure under the general condition that $\rho > 0$ and $p < 0$. This type of equation of state implies that the matter distribution is in tension and hence the matter is known, in the literature, as a ‘false vacuum’ or ‘degenerate vacuum’ or ‘ ρ -vacuum’ (Davies, 1984; Blome and Priester, 1984; Hogan, 1984; Kaiser and Stebbins, 1984). A natural question arises whether there exists any EMMM where this condition is violated, i.e., when $\rho + p \neq 0$. This is the main motivation of the present investigation and here we have shown that even for $\rho + p \neq 0$ EMMM can be constructed. However, to carry out this plan we have assumed here that the so-called cosmological constant is dependent on the space coordinate (Chen and Wu, 1990; Narlikar *et al.*, 1991; Ray and Ray, 1993; Tiwari and Ray, 1996). This will help us to have several class of solutions related to EMMM.

2. Einstein-Maxwell field equations

Let us consider a spherically symmetric line element

$$ds^2 = g_{ij}dx^i dx^j = e^{\nu(r)} dt^2 - e^{\lambda(r)} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (i, j = 0, 1, 2, 3). \quad (1)$$

Now, the Einstein field equations for the case of charged anisotropic source are

$$G^i_j = R^i_j - \frac{1}{2}g^i_j R + g^i_j \Lambda = -\kappa[T^i_j{}^{(m)} + T^i_j{}^{(em)}], \quad (2)$$

where $T^i_j{}^{(m)}$ and $T^i_j{}^{(em)}$ are respectively the energy-momentum tensor components for the anisotropic matter source and the electromagnetic field and are given by

$$T^i_j{}^{(m)} = (\rho + p_\perp)u^i u_j - p_\perp g^i_j + (p_\perp - p_r)\eta^i \eta_j \quad (3)$$

with $u_i u^i = -\eta_i \eta^i = 1$, and

$$T^i_j{}^{(em)} = \frac{1}{4\pi}[-F_{jk}F^{ik} + \frac{1}{4\pi}g^i_j F_{kl}F^{kl}]. \quad (4)$$

The Maxwell electromagnetic field equations are given by

$$[(-g)^{1/2}F^{ij}]_{,j} = 4\pi J^i(-g)^{1/2}, \quad (5)$$

and

$$F_{[ij,k]} = 0, \quad (6)$$

where the electromagnetic field tensor F_{ij} is related to the electromagnetic potentials through $F_{ij} = A_{i,j} - A_{j,i}$ which, obviously, is equivalent to the equation (5), viz., $F_{[i,j,k]} = 0$. Further, u^i is the 4-velocity of a fluid element, J^i is the 4-current satisfying $J^i = \sigma u^i$, where σ is the charge density, and $\kappa = 8\pi$ (in relativistic unit $G = C = 1$). Here and in what follows a comma denotes the partial derivative with respect to the coordinates (involving the index).

The Einstein-Maxwell field equations (2)–(6) corresponding to anisotropic charged source with cosmological variable, are then given by

$$e^{-\lambda}(\lambda'/r - 1/r^2) + 1/r^2 = 8\pi T^0_0 = 8\pi\rho + E^2 + \Lambda, \quad (7)$$

$$e^{-\lambda}(\nu'/r + 1/r^2) - 1/r^2 = -8\pi T^1_1 = 8\pi p_r - E^2 - \Lambda, \quad (8)$$

$$e^{-\lambda}[\nu'' + \nu'^2/4 - \nu'\lambda'/4 + (\nu' - \lambda')/2r] = -8\pi T^2_2 = -8\pi T^3_3 = 8\pi p_\perp + E^2 - \Lambda, \quad (9)$$

$$(r^2 E)'' = 4\pi r^2 \sigma e^{\lambda/2}. \quad (10)$$

The equation (10) can equivalently, in terms of the electric charge q , be expressed as

$$q(r) = r^2 E(r) = \int_0^r 4\pi r^2 \sigma e^{\lambda/2} dr \quad (11)$$

where p_r , p_\perp and E are the matter-energy density, radial and tangential pressures and intensity of the electric field respectively. Here prime denotes derivative with respect to radial coordinate r only.

The equation of continuity, $T^i_j; i = 0$ for the field equations (2) - (6), is given by

$$\frac{d}{dr} \left[p_r - (E^2 + \Lambda)/8\pi \right] + (\rho + p_r)\nu'/2 = E^2/2\pi r + 2(p_\perp - p_r)/r. \quad (12)$$

Now, we assume the relation between the radial and tangential pressure as

$$p_\perp = np_r, \quad (n \neq 1). \quad (13)$$

Assuming further that the radial stress $T^1_1 = 0$ (Florides, 1987; Kofinti, 1985; Grøn, 1986b; Ponce de Leon, 1987b; Tiwari and Ray, 1996), one gets

$$\nu' = (e^\lambda - 1)/r, \quad (14)$$

$$p_r = (E^2 + \Lambda)/8\pi. \quad (15)$$

Using equations (13) - (15), in equation (12), we get

$$\rho + p_r = [(n + 1)E^2 + (n - 1)\Lambda]/2\pi(e^\lambda - 1). \quad (16)$$

Similarly, equations (7) and (8) yield,

$$e^{-\lambda}(\nu' + \lambda') = 8\pi r(\rho + p_r). \quad (17)$$

Again, equation (7) together with equation (15), gives

$$e^{-\lambda} = 1 - 2m(r)/r, \quad (18)$$

where $m(r)$, called the gravitational mass, takes the form

$$m(r) = M(r) + \mu(r) = 4\pi \int_0^r [\rho + p_r]r^2 dr, \quad (19)$$

the Schwarzschild mass and the mass equivalence of electromagnetic field, respectively, being defined as

$$M(r) = 4\pi \int_0^r \rho r^2 dr, \quad \mu(r) = 4\pi \int_0^r p_r r^2 dr. \quad (20)$$

Now, from the above equation (19) it is easily observed that the condition $\rho + p_r = 0$, yields a flat space-time through the equations (18) and (14) and has been considered by us in another

context (Tiwari et al., 2000). Hence, the non-trivial solutions exists here for the case $\rho + p_r \neq 0$ only.

3. Solutions for the static charged fluid spheres

Let us now solve the equation (16) under the constraint $\rho + p_r \neq 0$, assuming different mathematical conditions. As the equation (16) is involved with two physical parameters so unless we specify one parameter it is not possible to solve the equation (19). Let us, therefore, assume the following two cases which will yield solutions with physically interesting features as the analysis in Section 4 demonstrates.

Case I:

We now make the choice

$$\Lambda = E^2 - N\Lambda_0, \quad (21)$$

where N is an integer and Λ_0 is the erstwhile non-zero cosmological constant.

With the help of equations (16) and (21), the equation (19) takes the form

$$m(r) = 2 \int_0^r [2nE^2 - (n-1)N\Lambda_0] r^2 dr / (e^\lambda - 1). \quad (22)$$

To make equation (22) integrable we assume that

$$E^2 = q^2/r^4 = [k(e^\lambda - 1)(1 - R^2) + (n-1)N\Lambda_0]/2n, \quad (23)$$

where k is a constant and $R = r/a$, a being the radius of the sphere. This particular choice for the electric intensity generates a model for charged sphere which is physically very interesting as it is related to EMMM as will be seen later on. Thus, the solution set is given by

$$e^{-\lambda} = 1 - AR^2(5 - 3R^2), \quad (24)$$

$$e^\nu = (1 - 2A)^{5/4} e^{\lambda/4} \exp[5B \tan^{-1} B(6R^2 - 5) - \tan^{-1} B/2], \quad (25)$$

$$p_r = p_\perp/n = [k(e^\lambda - 1)(1 - R^2) - N\Lambda_0]/8\pi n, \quad (26)$$

$$\rho = [k(1 + 4n - e^\lambda)(1 - R^2) + N\Lambda_0]/8\pi n, \quad (27)$$

where

$$A = 4ka^2/15, \quad B = [A/(12 - 25A)]^{1/2}. \quad (28)$$

Now, the exterior field of a spherically symmetric static charged fluid distribution described by the metric (1) is the unique Reissner-Nordström solution given by

$$ds^2 = \left[1 - \frac{2m}{r} + \frac{q^2}{r^2}\right] dt^2 - \left[1 - \frac{2m}{r} + \frac{q^2}{r^2}\right]^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (29)$$

Then, application of the matching condition on the boundary $r = a$ yields the total effective gravitational mass which is given by

$$m(a) = \frac{4}{15}ka^3 + \frac{q(a)^2}{2a}. \quad (30)$$

Hence, in terms of the total gravitational mass, the total electric charge and the radius of the sphere the constants k , A and B can be expressed as

$$k = 15(2am - q^2)/8a^4, \quad (31)$$

$$A = (2am - q^2)/2a^2, \quad (32)$$

and

$$B^2 = (2am - q^2)/[24a^2 - 25(2am - q^2)]. \quad (33)$$

Comparing the equation (19) (and hence via the equation (20)) with the above equation (30) we can easily recognize the first term $(4ka^3/15)$ in the right hand side as the total Schwarzschild mass whereas the second term $(q^2/2a)$ is the total mass equivalence of the electromagnetic field. Now, k , in the first term of equation (30), is implicitly related to the charge q as is evident from the equation (23). Therefore, in principle, the gravitational mass m is of purely electromagnetic origin.

Case II:

Here the choice is

$$\Lambda = N\Lambda_0 - E^2. \quad (34)$$

Then, from the equations (16) and (34), the gravitational mass (equation (19)) reduces to

$$m(r) = 2 \int_0^r [2E^2 + (n-1)N\Lambda_0] r^2 dr / (e^\lambda - 1). \quad (35)$$

For the further assumption

$$E^2 = q^2/r^4 = [k(e^\lambda - 1)(1 - R^2) - (n-1)N\Lambda_0]/2, \quad (36)$$

we have the solution set

$$e^{-\lambda} = 1 - AR^2(5 - 3R^2), \quad (37)$$

$$e^\nu = (1 - 2A)^{5/4} e^{\lambda/4} \exp[5B \tan^{-1} B(6R^2 - 5) - \tan^{-1} B/2], \quad (38)$$

$$p = N\Lambda_0/8\pi, \quad (39)$$

$$\rho = [4k(1 - R^2) - N\Lambda_0]/8\pi. \quad (40)$$

We see that λ and ν retain the same form as in case I and hence the total gravitational mass is also given by the equation (30). Further, it is observed that the present case automatically reduces to an isotropic one as the pressure p does not associated with the anisotropic factor n .

4. Physical properties of the static charged fluid spheres

Case I:

(1) The equation (23) indicates that for getting a direct dependence of k upon q one can admit the following relaxation such that (i) $N = 0$ when $\Lambda_0 \neq 0$ and $(n - 1) \neq 0$, (ii) $(n - 1) = 0$ when $N \neq 0$ and $\Lambda_0 \neq 0$ and (iii) $\Lambda_0 = 0$ when $(n - 1) \neq 0$ and $N \neq 0$. The third possibility seems to contradict the observational results related to the Supernova type Ia where the cosmological constant is found to be a non-zero value whereas the second one provides an isotropic case. Then, suitably opting for the sub-case (i), viz., $N = 0$ one obtains

$$q^2 = kr^4(e^\lambda - 1)(1 - R^2)/2n, \quad (41)$$

$$p_r = p_\perp/n = k(e^\lambda - 1)(1 - R^2)/8\pi n, \quad (42)$$

$$\rho = k(1 + 4n - e^\lambda)(1 - R^2)/8\pi n. \quad (43)$$

Thus, for vanishing electric charge the gravitational mass in the equation (30), including all the physical parameters (viz., pressures and density), vanishes and one obtains EMMM.

(2) The central and the boundary pressures are found to be equal here i.e., $p_r(0) = p_\perp(0)/n = -N\Lambda_0/8\pi n$ and $p_r(0) = p_\perp(0)/n = -N\Lambda_0/8\pi n$ respectively whereas the respective densities are $(4nk + N\Lambda_0)/8\pi n$ and $N\Lambda_0/8\pi n$. So, the present model has a constant pressure throughout the sphere though the density decreases from center to boundary. For the value $N = 0$, however, we have zero pressure, both at the center and boundary, and the density decreases from the central value $k/2\pi$ to zero at the boundary. Thus, with $N = 0$ the present model goes to a physically well-behaved static charged dust case. However, we would like to investigate how the pressure and the density behave in between the center to boundary. To see this, as an example,

we have plotted the graph for the extended classical electron of Lorentz type (1904) in Figure 1. It shows the nature of variation of the fluid pressure and the energy density with radius from the center of the matter distribution to the boundary. We observe that though the pressure is zero at the center as well as at the boundary but the in-between feature is very interesting. Starting from zero it increasing slowly and then after attaining a maximum value drops steadily to zero. In the case of density the curve starts from the non-zero central value $k/2\pi$ and then smoothly decreases to zero at the boundary. However, it is to be noted here that for the electron with radius $a = 10^{-16}$ cm, mass $m = 6.76 \times 10^{-56}$ cm and charge $q = 1.38 \times 10^{-34}$ the value of the constant k becomes a negative quantity with -3.568215×10^{-4} . This particular aspect one should keep in mind for the explanation of the internal structure of classical electron with respect to the fluid pressure and the energy density. Thus we see that the density starts from the down of the axis due to this negative value attached to it. In the Figure 2 the pressure-density profile is shown for different values of the anisotropic factor n .

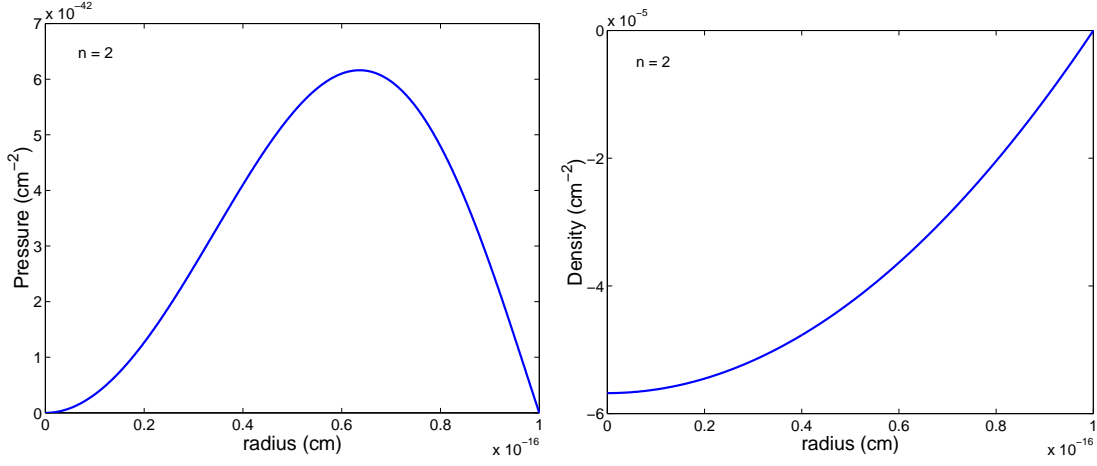


Figure 1: The fluid pressure and the energy density as a function of radius are plotted for a classical electron with radius $a = 10^{-16}$ cm, mass $m = 6.76 \times 10^{-56}$ cm and charge $q = 1.38 \times 10^{-34}$ cm.

(3) In the above analysis for the general value of N we have seen that the fluid pressure is altogether negative whereas the density is a positive quantity. Now, it can be observed from the equations (26) and (27) that $\rho + p_r \neq 0$ except at the boundary where it is equal to $p_r(a) = -\rho(a)$, where $\rho(a) > 0$ and $p(a) < 0$ as seen earlier. Otherwise, it will have a general value $k(1 - R^2)/2\pi$

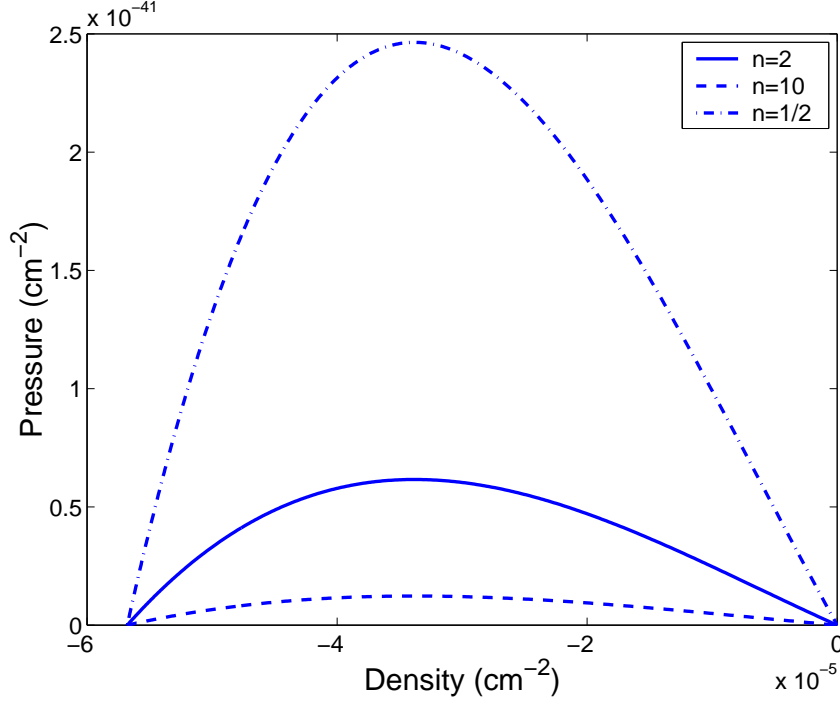


Figure 2: The fluid pressure and the energy density profile is shown for a classical electron with radius $a = 10^{-16}$ cm, mass $m = 6.76 \times 10^{-56}$ cm and charge $q = 1.38 \times 10^{-34}$ cm.

. The central value is then $p_r(0) = -\tilde{\rho}(0)$ where $\tilde{\rho}(0) = \rho(0) - k/2\pi$. As the model demands for $\rho > 0$ and $p < 0$ so the condition to be satisfied here is $\rho(0) > k/2\pi$. The general condition for the negative pressure and positive density is then $\rho > k(1 - R^2)/2\pi$ for all $r \leq a$. These results are also true for the sub-case $N = 0$.

Case II:

(1) Here for $N = 0$ the solution set regarding the pressure and density (vide equations (39) and (40)) reduces to

$$p = 0, \quad (44)$$

$$\rho = k(1 - R^2)/2\pi, \quad (45)$$

when the electric charge is given by

$$q^2 = k(e^\lambda - 1)(1 - R^2)r^4/2. \quad (46)$$

Thus, as in the previous case, for $q = 0$ we get $k = 0$ which in turn makes mass and density

everything zero and also the space-time becomes flat. Thus, the model presented here is an EMMM.

(2) In the present case also the central and the boundary pressures are equal with a value $N\Lambda_0/8\pi$ and the respective densities are $(4k - N\Lambda_0)/8\pi$ and $-N\Lambda_0/8\pi$. The pressures become zero for $N = 0$, both at the center and boundary, and densities have the positive central value $k/2\pi$ whereas the boundary-value is zero. Thus, we again get a physically interesting charged dust case with $N = 0$, only difference with the previous Case I is that this is now a case of isotropic fluid sphere. Figure 3 shows the nature of variation of energy density with radius from the center of the matter distribution to the boundary for an extended classical electron. Here also the behavior is regular and well defined.

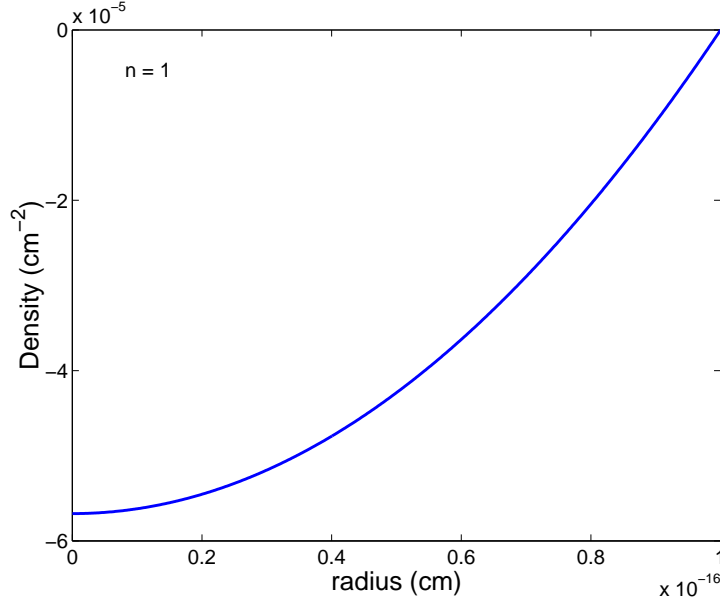


Figure 3: The energy density as a function of radius is plotted for a classical electron with radius $a = 10^{-16}$ cm, mass $m = 6.76 \times 10^{-56}$ cm and charge $q = 1.38 \times 10^{-34}$.

(3) In the present case also, by virtue of equations (39) and (40), $\rho + p_r \neq 0$ which reads here as $\rho + p_r = k(1 - R^2)/2\pi$. Here the central value, at $r = 0$, is $\rho = -\tilde{p}$ where $\tilde{p} = (p - k/2\pi)$ and the boundary one is $\rho = -p$. Due to negativity of the density here the condition on the pressure to be imposed is $p > k/2\pi$. Thus, the present Case II clearly provides an EMMM even with a positive pressure and therefore contradicts the comment made by Ivanov (2002) that

“... electromagnetic mass models all seem to have negative pressure.” The same result, i.e. the positivity of pressures are also available in some cases of the work done by Ray and Das (2002) related to EMMM. However, the explanation given here is valid for any positive value of N and so the situation may completely be opposite with any negative value of N . At this stage, we feel, we should not put any restriction on the choice of the value of N . This is because, in general, for a fluid sphere we should have $p \geq 0$ and $\rho \geq 0$ so that the weak energy conditions are satisfied. But there are also some special situations available within the spherical system (particularly, in the case of electron with the radius $\sim 10^{-16}$ cm) where the energy condition is violated due to negative energy density (Cooperstock and Rosen, 1989; Bonnor and Cooperstock, 1989). Thus, choosing the proper signatures of N we can have a class of models with diverse characters.

5. Role of Λ : Previous and Present Status

The cosmological constant was introduced by Einstein in his field equation to obtain a static cosmological solution because of the fact that due to gravitational pull everything will collapse to a point and hence a un-wanting situation of singularity will take place. However, he was not satisfied with this new physical quantity as it seemed to violate Machian principle which he tried to incorporate in the framework of his general theory of relativity. He thus, ultimately rejected it mainly for two reasons: (i) that the theoretical work of de Sitter showing that the Einstein’s field equations admitted a solution for empty Universe and (ii) that the experimental discovery of expanding Universe by Hubble.

As stated in the introduction, the concept of cosmological constant has been revived recently in the case of early Universe scenario and even in particle physics. It is gradually being felt that Λ , the erstwhile cosmological constant is available rather than a constant, as was being believed earlier, varying with space or time or both (Ray and Ray, 1993; Tiwari and Ray, 1996; Tiwari et al., 2000). Further, Λ may be positive or negative (by imposing the condition that its value is not equal to zero). For instance, according to Zel’dovich (1968) the effective gravitational mass density of the polarized vacuum is negative. Similarly, the equation of state $\rho + p = 0$, employed by Tiwari et al. (1984) to construct EMMM as a solution of Einstein-Maxwell field equations, provides negative pressure. It may be emphasized here that positive density has significant, rather major role in inflationary cosmology whereas negative density has influence on elementary particle models. The gravitational mass inside the spherical charged body is negative for

$r < 5a/4$, where r is the radial coordinate and a is the radius of the sphere (Gautreau, 1985). It is argued by Grøn (1986a, b) that this negative mass and the associated gravitational repulsion is due to the strain of the vacuum because of vacuum polarization. He also argue that if a vacuum has a vanishing energy, then its gravitational mass will be negative and the observed expansion of the universe may be explained as a result of repulsive gravitation. Now, if we consider a negative Λ having a repulsive nature as was considered by Einstein then this gets the same status of negative pressure and also can be identified with the Poincare' stress. This repulsive gravitation associated with negative Λ can also be explained as the source of gravitational blue shift (Grøn, 1986a). On the contrary, positive Λ will be related to gravitational red shift. It may be also pointed out that according to Ipser and Sikivie (1984) domain walls are sources of repulsive gravitation and a spherical domain wall will collapse. To overcome this situation the charged "bubbles" with negative mass keep the wall static and hence in equilibrium. In this regard, we may also add that Λ , via repulsive gravitation, is related to domain walls and playing an important physical role.

Very recent observations conducted by the SCP and HZT (Perlmutter *et al.*, 1998; Riess *et al.*, 1998; Filippenko, 2001; Kastor and Traschen, 2002) show that the present value of Λ is positive one and hence related to the repulsive pressure. It is believed that the present state of acceleration dominated universe is due to the driven force of this Λ . It is to be noted that the negative Λ corresponds to a collapsing situation of the universe (Cardenas *et al.*, 2002).

6. Conclusions

(i) In both the above cases I and II, it is possible to show that EMMM also can be obtained, in principle, using the constraint $\rho + p_r \neq 0$. This particular point remained unnoticed by Grøn (1986a, b) and Ponce de Leon (1987a, b), both. However, as $\rho + p_r = 0$ is related to vacuum polarization, black hole physics and inflationary cosmology (Grøn, 1985, 1986a; Wenda and Shitong, 1985a, b; Guth, 1981; Linde, 1984), so the present model, in general, is in contradiction to those phenomenological explanations.

(ii) It can be noted that in terms of energy-momentum tensor of the fluid the condition $\rho + p_r = 0$ implies $T^1_1 = T^0_0$ (Herrera and Varela, 1996) whereas $\rho + p_r \neq 0$ constraint may be expressed as $T^1_1 = 0$ as we have adopted in the present approach. It is also interesting to note that $\rho + p_r = 0$ and hence $T^1_1 = T^0_0$ can be expressed in terms of the metric tensors (vide equation

(1)) as $g_{00}g_{11} = -1$. A coordinate-independent statement of this relation is obtained by Tiwari *et al.* (1984) by using the eigen values of the Einstein tensor G^i_j .

(iii) Another point is that in both the cases we have taken the assumptions in a way so that the cosmological variable Λ does not vanish rather may be at most equal to Λ_0 , the erstwhile cosmological constant having a finite non-zero value. Without considering Λ_0 we will have $\Lambda = 0$ at the boundary $r = a$ which is a bit unphysical and may create difficulties, such as to entropy like problems (Beesham, 1993). In this regard, one can observe that the solutions obtained by Grøn(1986a, b) and Ponce de Leon (1987a, b) represent a neutral system, viz., though the net charge is not zero but the charge on the surface of the spherical system vanishes. The models of the present paper, in general, do not correspond to this situation because of the fact that the cosmological parameter Λ does not vanish at the boundary, rather, its value is Λ_0 at $r = a$. Therefore, the present solutions correspond to charged sphere. Of course, for $N = 0$, like Grøn (1986, b) and Ponce de Leon (1987a, b), we have neutral spheres (vide equation (41) of case I and equation (46) of case II). Thus, we have a class of solutions related to charged as well as neutral systems depending on the values of N .

(iv) We have, as a special case, applied the present models to the classical electron of Lorentz type. However, it seems that a more realistic application of the models with variable cosmological constant is possible in the case of charged massive astrophysical systems, like neutron stars. This aspect will be carried out in detail in the future investigations.

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